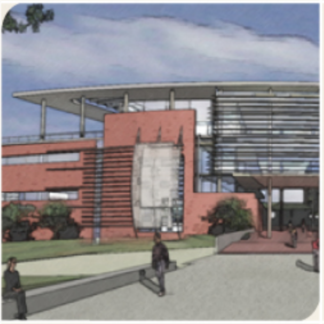
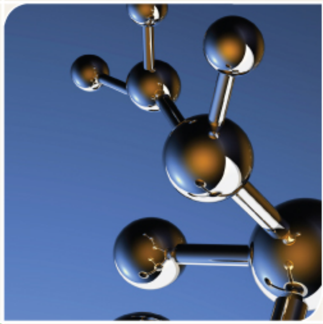
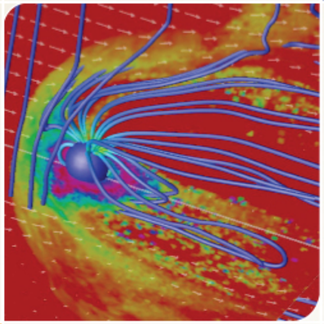
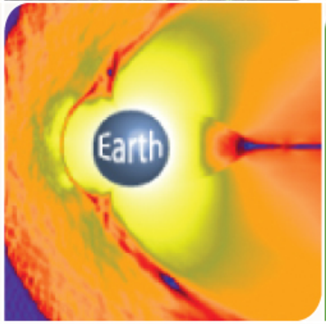
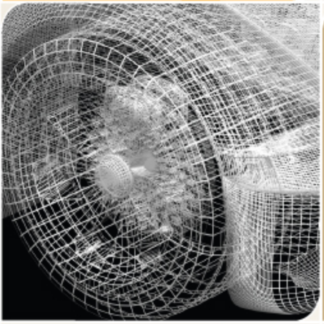








Scalable K-Core Decomposition for Static Graphs Using a Dynamic Graph Data Structure



Alok Tripathy

Georgia Tech College of Computing
Computational Science and Engineering

What I'll Show

- Maximal k-core algorithm
  - Up to $4X$ faster than previous research
  - Up to $58X$ faster than popular graph libraries
- k-core edge decomposition algorithm
  - Up to $8X$ faster than previous research
  - Up to $129X$ faster than popular graph libraries

What I'll Show

- Maximal k-core algorithm
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- k-core edge decomposition algorithm
  - Up to $8X$ faster than previous research
  - Up to $129X$ faster than popular graph libraries

  - Uses a dynamic graph operations

Takeaways

- Algorithms on static graphs can use dynamic graph operations efficiently with the GPU.
- Dynamic graph operations can be computed on a GPU efficiently.
  - Check out the Hornet data structure!
  - https://github.com/hornet-gt/hornet

Motivation

- Two types of graphs
  - Static graphs that don't change
  - Dynamic graphs that change frequently
    - Edge/vertex insertions/deletions
    - e.g. Facebook, road networks

Motivation

- Two types of graphs
  - Static graphs that don't change
  - Dynamic graphs that change frequently
    - Edge/vertex insertions/deletions
    - e.g. Facebook, road networks

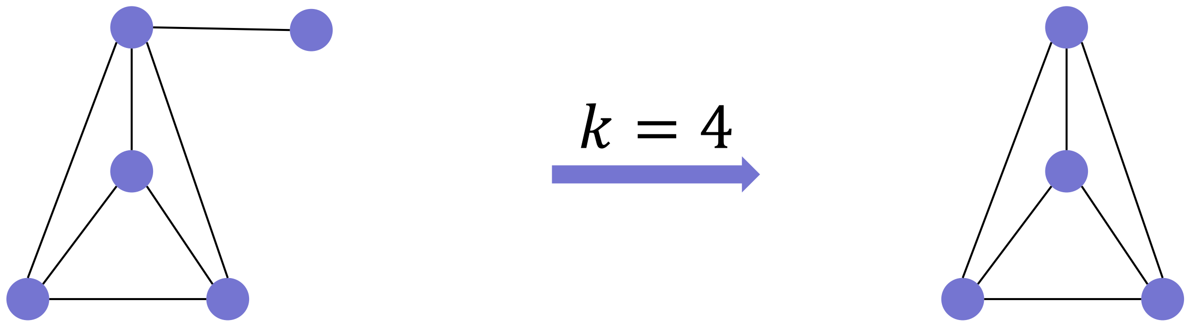
- Algorithms on static graphs can benefit from dynamic graph operations

Dynamic Operations on Static Graphs

- k-truss problem

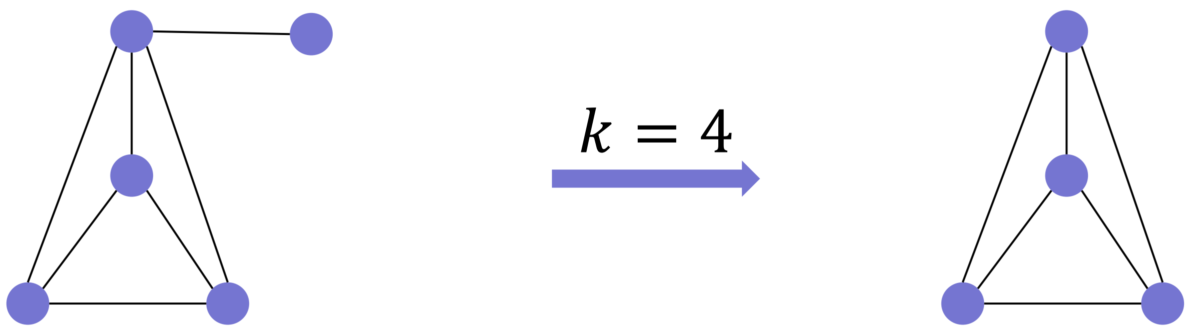
Dynamic Operations on Static Graphs

- k-truss problem
  - Subgraph where all edges belong to at least $k - 2$ triangles
  - Can be extended to maximal k-truss



Dynamic Operations on Static Graphs

- k-truss problem
  - Subgraph where all edges belong to at least $k - 2$ triangles
  - Can be extended to maximal k-truss
  - Applications: community detection, anomaly detection



k-truss Algorithm

- $E_m =$ all edges in $\geq k - 2$ triangles
- while $|E_m| > 0$
  - delete E_m from G
  - update triangles in G
  - $E_m =$ all edges in $\geq k - 2$ triangles

Takeaways

- Algorithms on static graphs can use dynamic graph operations efficiently with the GPU.
- Dynamic graph operations can be computed on a GPU efficiently.
  - Check out the Hornet data structure!
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Widely used graph data structures

Names	Pros	Cons
Dense Adjacency Matrix	• Supports updates	• Poor locality • Massive storage requirements
Linked lists	• Flexible	• Poor locality • Limited parallelism • Allocation time is costly
COO (Edge list) - unsorted	• Has some flexibility • Updates are simple • Lots of parallelism	• Poor locality • Stores both the source and destination
CSR	• Uses exact amount of memory • Good locality • Lots of parallelism	• Inflexible

These data structures don't cut it

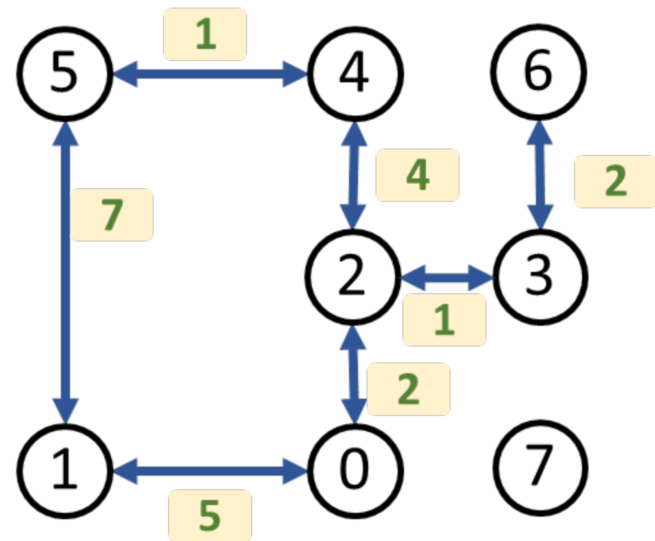
Compressed Sparse Row (CSR)

Pros:

- Uses precise storage requirements
- Great locality
  - Good for GPUs
- Handful of arrays
  - Simple to use and manage

Cons:

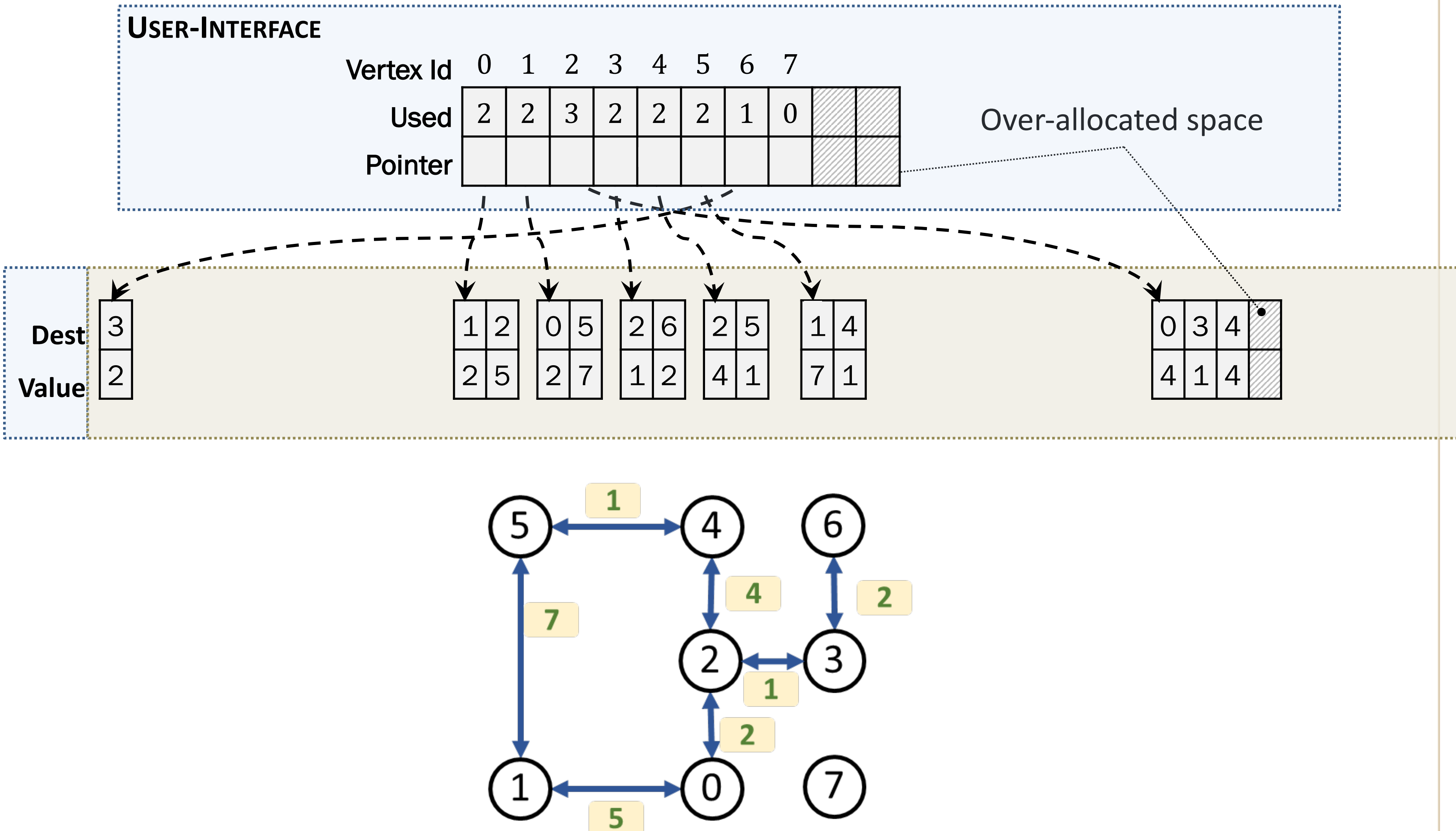
- Inflexible.
- Network growth unsupported
- Topology changes unsupported
- Property graphs not supported



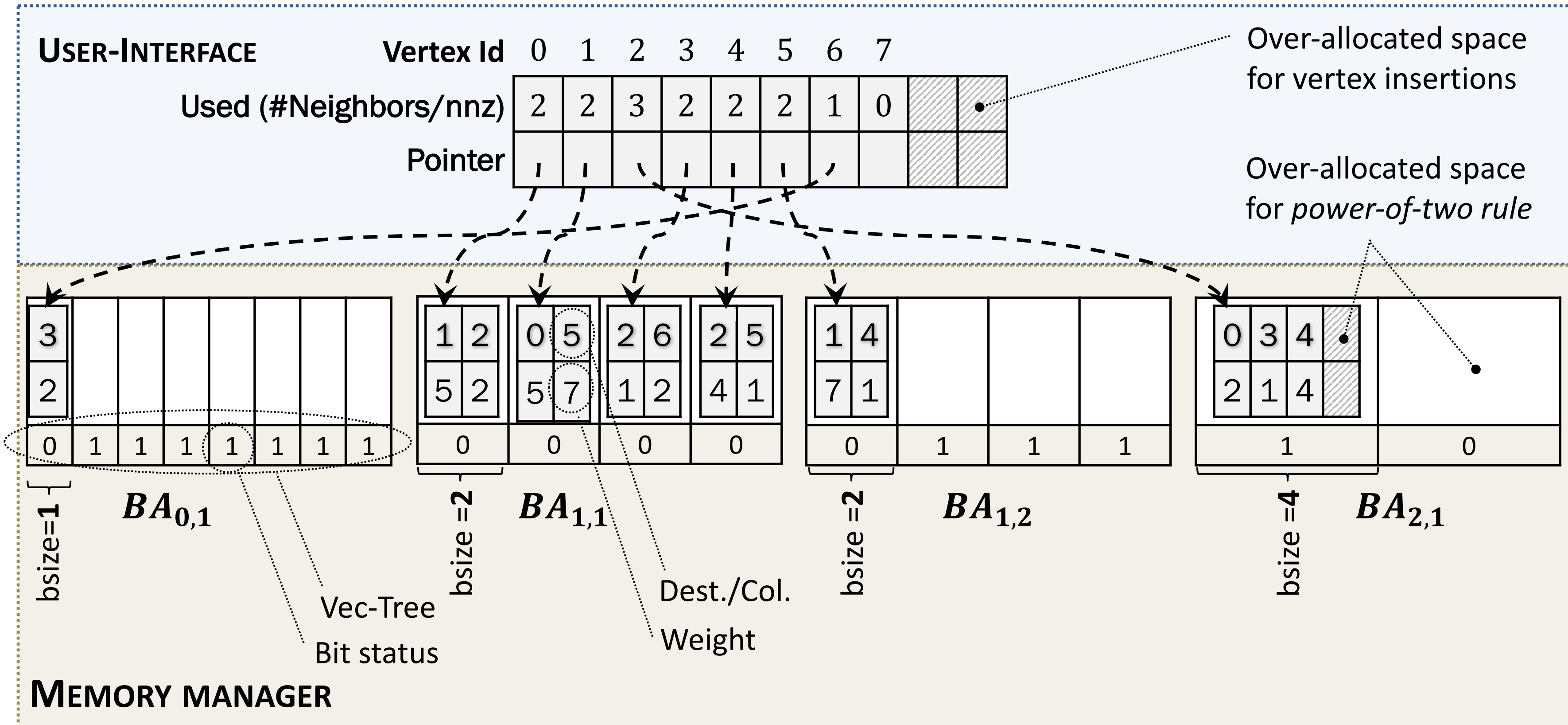
Src/Row	0	1	2	3	4	5	6	7	
Offset	0	2	4	7	9	11	13	14	14

Dest./Col.	1	2	0	5	0	3	4	2	6	2	5	1	4	3
Value	2	5	2	7	4	1	4	1	2	4	1	7	1	2

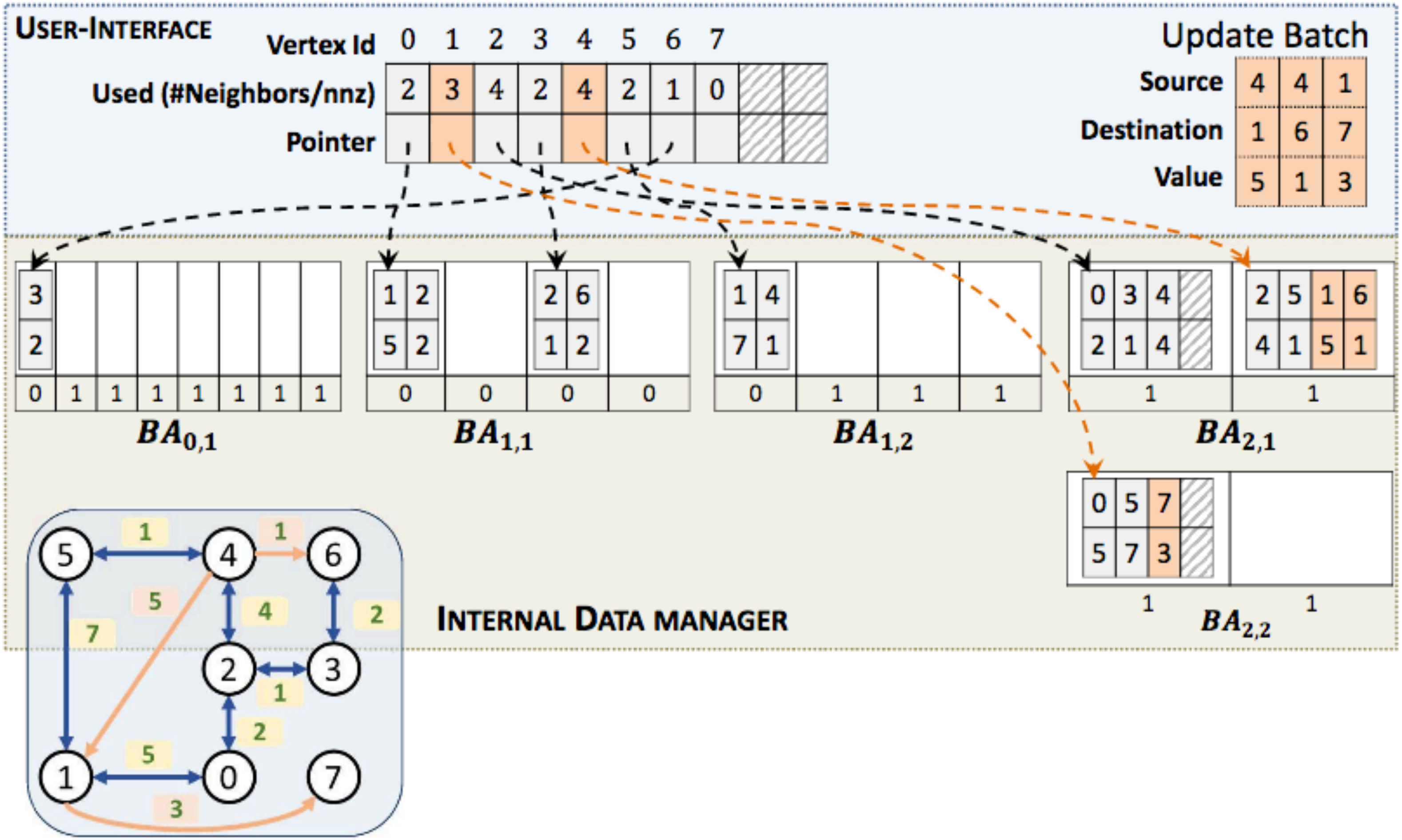
Hornet – A High Level View



Hornet in Detail



Hornet Insertion



(b) The updated graph.

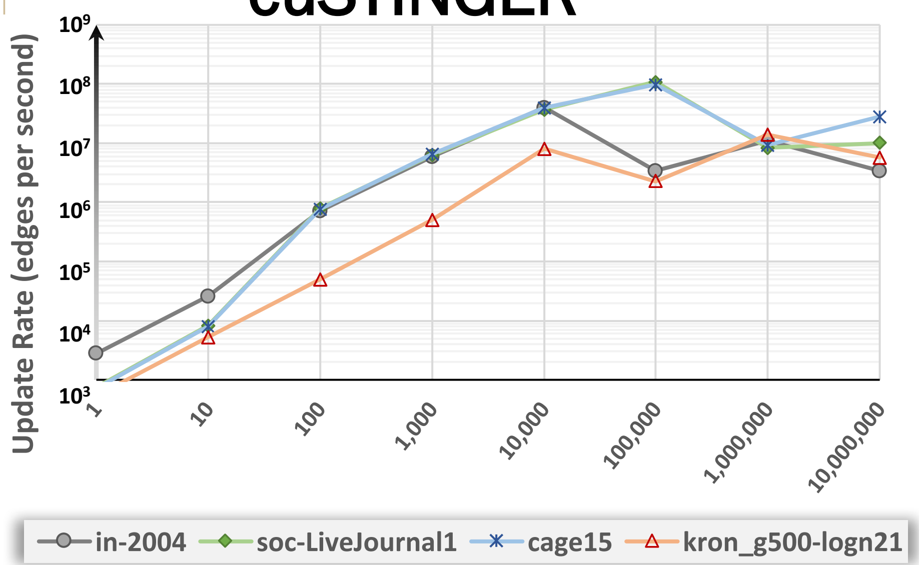
Hornet Insertion Pseudocode

```
parallel for (u, v) in batch
  - if u’s block is too full
     - allocate a new block
     - queue.add(u)

parallel for v in queue
  - copy adjacency list to new block

parallel for (u, v) in batch
  - add (u, v) to u’s block
```

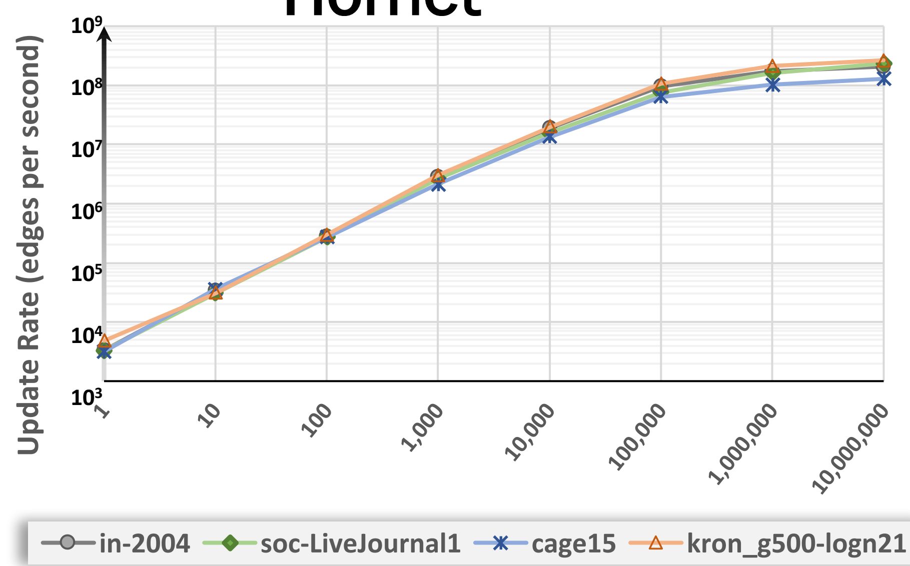
Insertion Rates

- Supports over 150M updates per second
- Hornet
  - $4X - 10X$ faster than cuSTINGER
  - Does not have $performance\ dip$ like cuSTINGER
- Scalable growth in update rate

cuSTINGER



Hornet



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- Current idea:
  - Dynamic graph operations are only for dynamic graphs, not static graphs.
    - Very expensive
    - Why bother?

Motivation

- Current idea:
  - Dynamic graph operations are only for dynamic graphs, not static graphs.
    - Very expensive
    - Why bother?
- New idea: Algorithms on static graphs can benefit from dynamic graph operations
  - If we can efficiently parallelize operations

What I'll Show

- 3 static graph algorithms
  - All 3 leverage NVIDIA P100 GPUs.
    - 2 beat the state-of-the-art
    - 1 does not (does not have good GPU utilization)

Algorithms

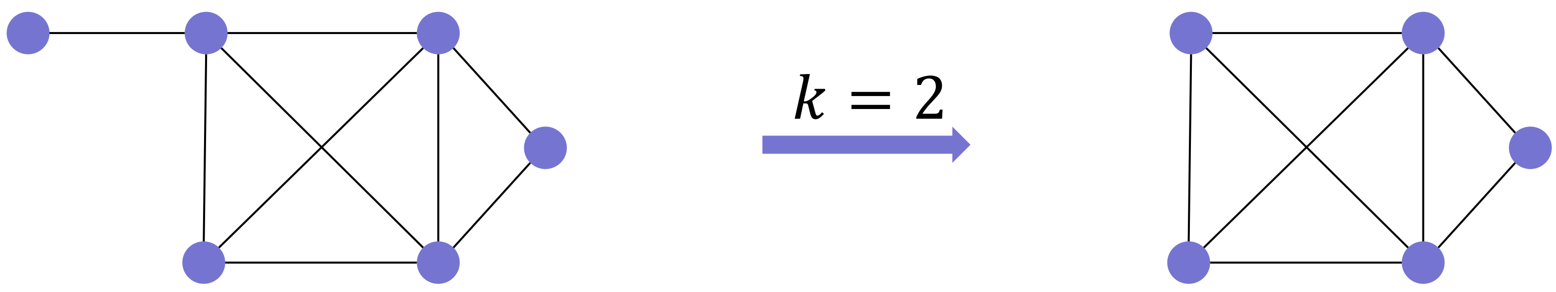
- Old maximal k-core algorithm
- New maximal k-core algorithm
- k-core edge decomposition

Algorithms

- Old maximal k-core algorithm ☹
- New maximal k-core algorithm
- k-core edge decomposition

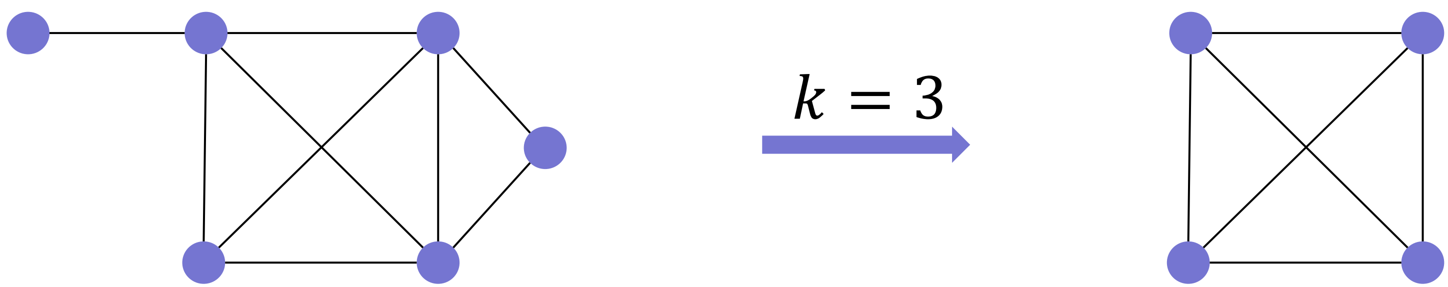
Maximal k-core Definitions

- k-core
  - Maximal subgraph where all vertices have degree at least k



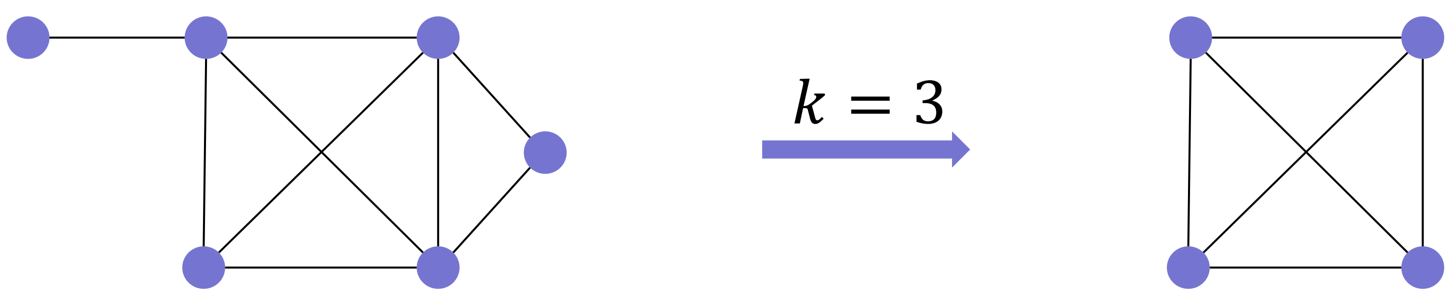
Maximal k-core Definitions

- k-core
  - Maximal subgraph where all vertices have degree at least k
- Maximal k-core
  - Largest k such that k-core exists in graph

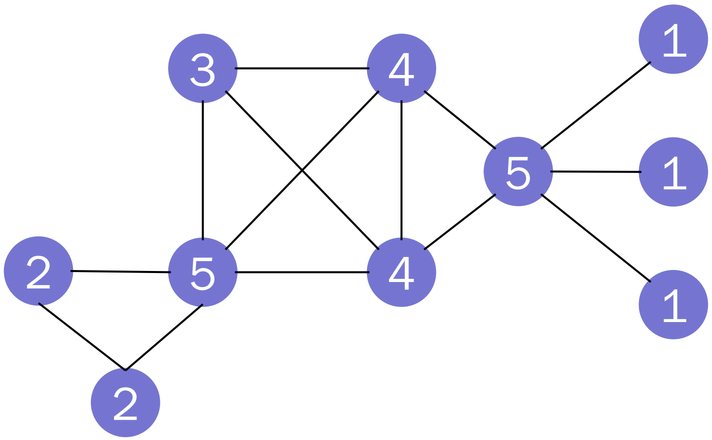


Maximal k-core Definitions

- k-core
  - Maximal subgraph where all vertices have degree at least k
- Maximal k-core
  - Largest k such that k-core exists in graph
- Applications: visualization, community detection



Maximal k-core High-Level



$peel = 1$

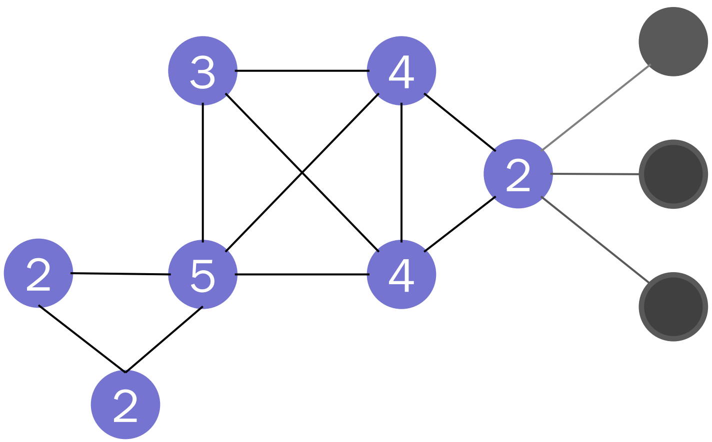
$peel = 0$

```
while vertices exist in G

    - delete all vertices
      with degree <= peel

    - if there aren't any
        - increment peel
```

Maximal k-core High-Level



$peel = 2$

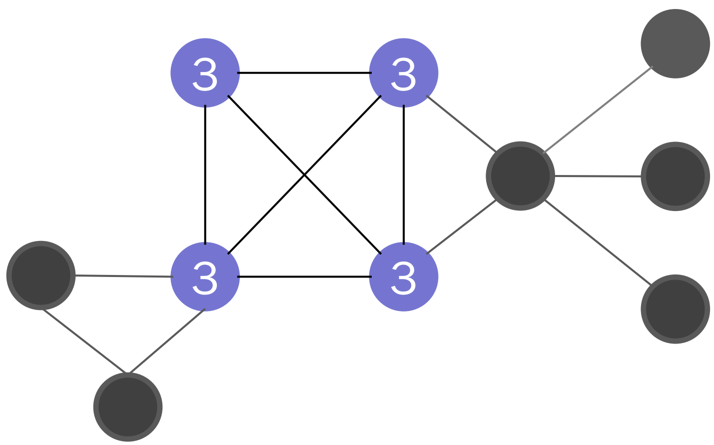
$peel = 0$

```
while vertices exist in G

   - delete all vertices
     with degree <= peel

   - if there aren't any
       - increment peel
```

Maximal k-core High-Level



$peel = 3$

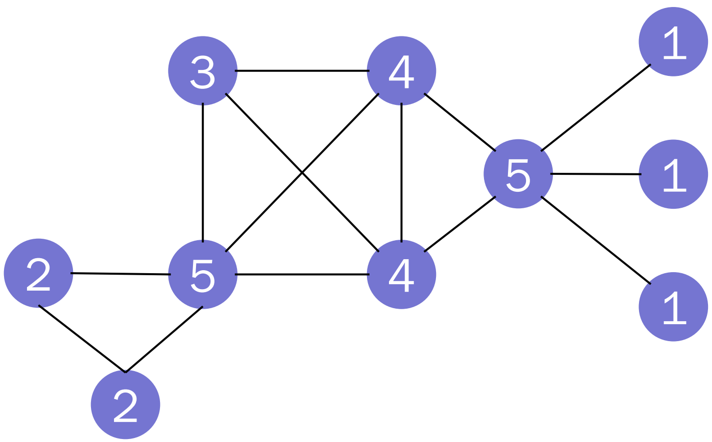
$peel = 0$

```
while vertices exist in G

   - delete all vertices
     with degree <= peel

   - if there aren't any
       - increment peel
```

Old Maximal k-core Algorithm



$peel = 1$

$peel = 0$
while vertices exist in G
- reset colors
- color all vertices with degree $\leq peel$
- if #coloredvertices > 0
  - delete colored vertices
  - delete incident edges
  - insert vertices in $\hat{G}$
  - insert edges in $\hat{G}$
- else
  - increment $peel$

Old Maximal k-core Code

```
while (nv > 0) {
    forAllVertices(hornet, SetColor { vertex_color });
    forAllVertices(hornet, CheckDeg { vqueue, peel_vqueue, vertex_pres, vertex_color,
                  peel });

    vqueue.swap();
    nv -= vqueue.size();


    if (vqueue.size() > 0) {
        gpu::memsetZero(hd().counter);

        forAllEdges(hornet, vqueue, PeelVertices { hd, vertex_color }, load_balancing);

        cudaMemcpy(&size, hd().counter, sizeof(int), cudaMemcpyDeviceToHost);

        if (size > 0) {
            oper_bidirect_batch(hornet, hd().src, hd().dst, size, DELETE);
            oper_bidirect_batch(h_copy, hd().src, hd().dst, size, INSERT);
        }

        *ne -= size;

        vqueue.clear();
    } else {
        peel++;
        peel_vqueue.swap();
    }
}
*max_peel = peel;
```

Compared Against

- ParK
  - parallel k-core algorithm; IEEE BigData 2014
  - Some parallelism
  - No dynamic graph operations
- igraph
  - network analysis toolkit
  - Sequential
  - No dynamic graph operations
- Both run on Intel Xeon E5-2695; 36 cores, 72 threads

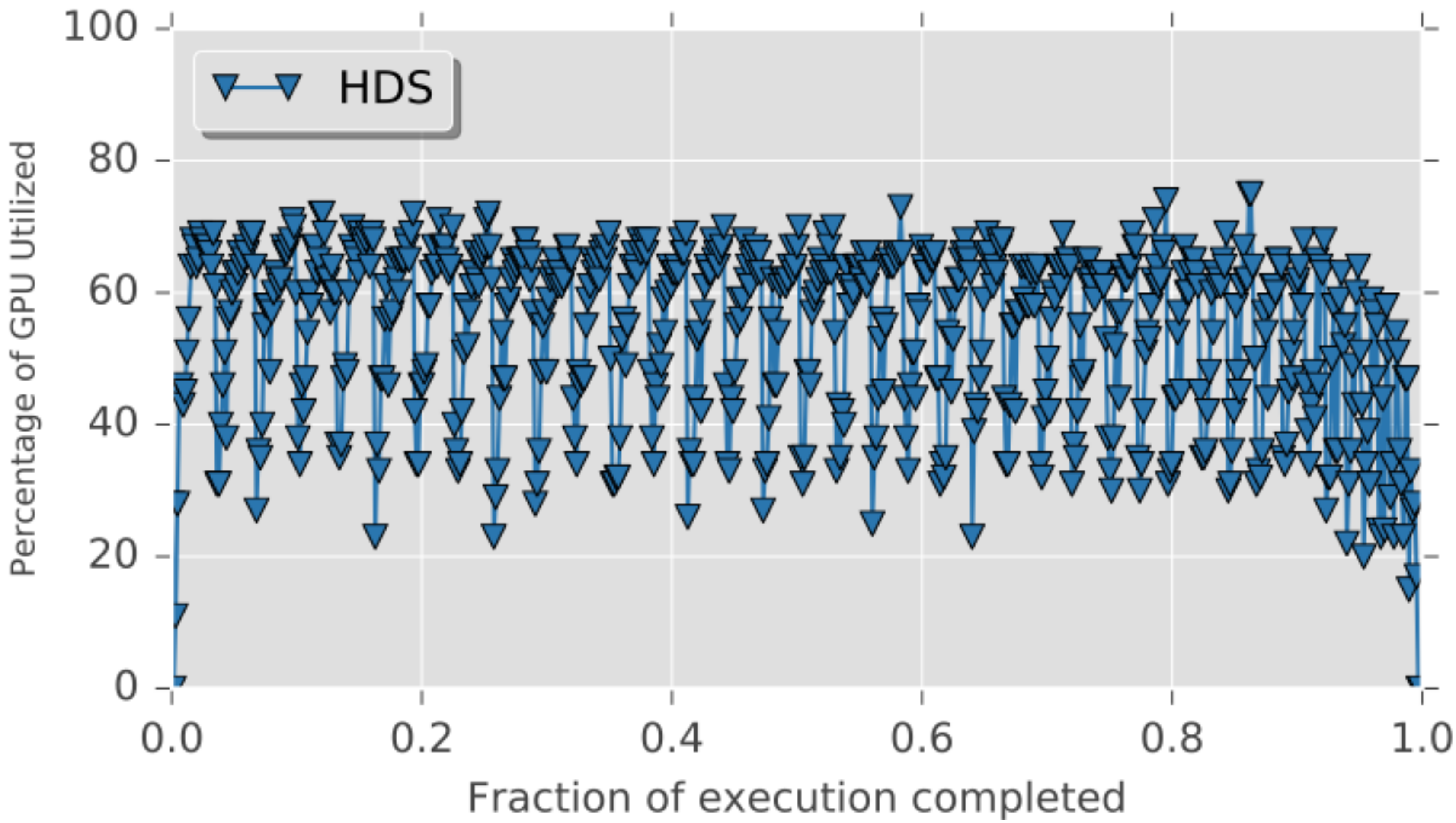
Old Maximal k-core Results

- Our algorithm is sometimes better than igraph.
- Our algorithm never beats ParK.

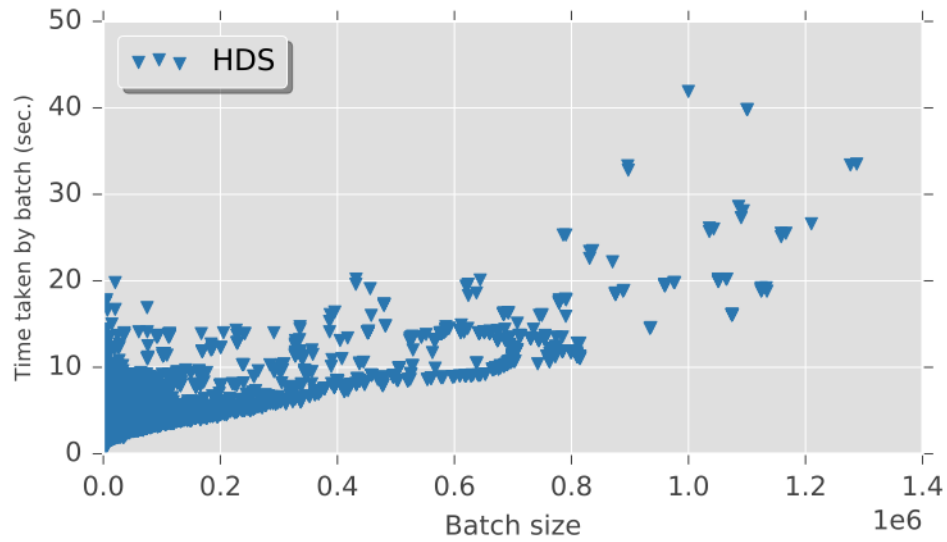
- Why are we so slow?

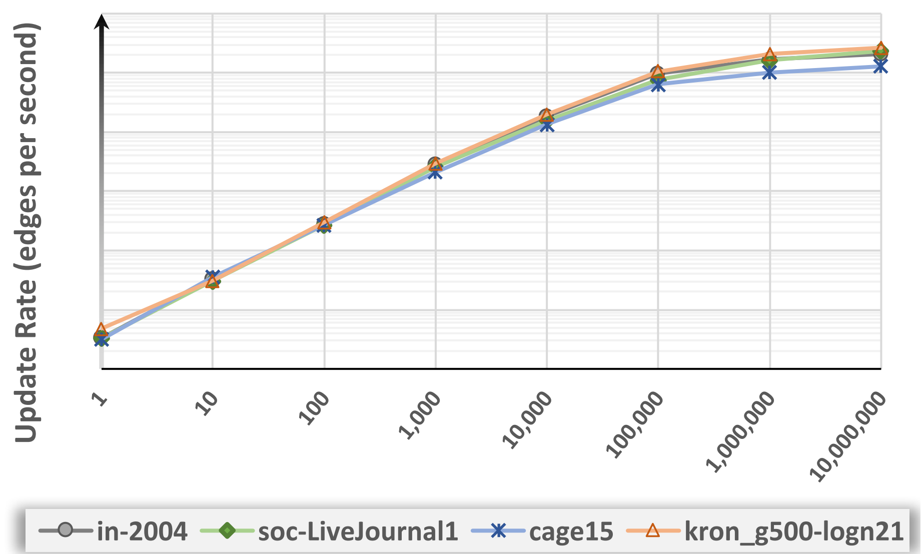
$Name$	$\lvert V \rvert$	$\lvert E \rvert$	$Our\ algorithm$	$ParK$	$igraph$
$dblp - author$	$5.5M$	$8.6M$	$2.2X$	$15X$	$1X$
$patentcite$	$3.8M$	$16.5M$	$1.3X$	$15X$	$1X$
$soc - LiveJournal1$	$4.8M$	$42.9M$	OOM	$11.3X$	$1X$
$soc - pokec - relationships$	$1.6M$	$22.3M$	$0.6X$	$16.6X$	$1X$
$trackers$	$27.7M$	$140.6M$	OOM	$6.8X$	$1X$
$wikipedia - link - de$	$3.2M$	$65.8M$	OOM	$5.1X$	$1X$

GPU Utilization



GPU Utilization / Batch Size





Algorithms

- Old maximal k-core algorithm ☹
- New maximal k-core algorithm
- k-core edge decomposition

Algorithms

- Old maximal k-core algorithm ☹
- New maximal k-core algorithm ☺
- k-core edge decomposition

New Maximal k-core Algorithm

- Flag vertices instead of deleting them.

```
while not every vertex is flagged

    - flag all vertices with degree <= peel

    - if there aren't any
        - increment peel
    - else
        - for each flagged vertex v
            - for each neighbor of v
                - decrement neighbor's degree
```

New Maximal k-core Code

```
int n_active = active_queue.size();
uint32_t peel = 0;

while (n_active > 0) {
    forAllVertices(hornet, active_queue,
            PeelVerticesNew { vertex_pres, deg, peel, peel_queue, iter_queue} );
    iter_queue.swap();

    n_active -= iter_queue.size();

    if (iter_queue.size() == 0) {
        peel++;
        peel_queue.swap();
        if (n_active > 0) {
            forAllVertices(hornet, active_queue, RemovePres { vertex_pres });
        }
    } else {
        forAllEdges(hornet, iter_queue, DecrementDegree { deg }, load_balancing);
    }
}
```

New Maximal k-core Results

- Our algorithm always beats igraph.
- Our algorithm is sometimes better than ParK.
  - At best, $3.9X$ faster
  - At worst, $4.3X$ slower
- Learned that batch size affected performance.

$Name$	$\lvert V \rvert$	$\lvert E \rvert$	$Our\ algorithm$	$ParK$	$igraph$
$dblp - author$	$5.5M$	$8.6M$	$58X$	$15X$	$1X$
$patentcite$	$3.8M$	$16.5M$	$26X$	$15X$	$1X$
$soc - LiveJournal1$	$4.8M$	$42.9M$	$7.4X$	$11.3X$	$1X$
$soc - pokec - relationships$	$1.6M$	$22.3M$	$15X$	$16.6X$	$1X$
$trackers$	$27.7M$	$140.6M$	$1.6X$	$6.8X$	$1X$

Algorithms

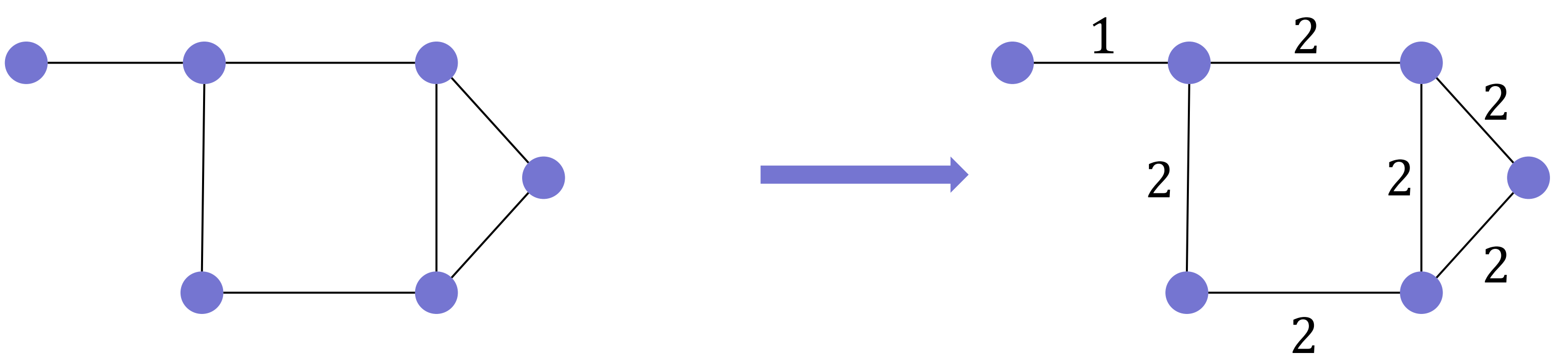
- Old maximal k-core algorithm ☹
- New maximal k-core algorithm ☺
- k-core edge decomposition

Algorithms

- Old maximal k-core algorithm ☹
- New maximal k-core algorithm ☺
- k-core edge decomposition ☺

k-core Decomp. Definitions

- k-core edge decomposition
  - For each edge, what is the largest k-core that edge belongs to?



k-core Decomp. Algorithm

```
while vertices exist in G
    - find the maximal k-core in G
    - mark all edges in k-core with value
      k
    - delete k-core from G
```

k-core Decomp. Code

```
while (peel_edges < ne) {
    uint32_t max_peel = 0;
    int batch_size = 0;

    maximal_kcore(hornet, hd_data, peel_vqueue, active_queue, iter_queue,
                  load_balancing, vertex_deg, vertex_pres, &max_peel, &batch_size);

    if (batch_size > 0) {
        cudaMemcpy(src + peel_edges, hd_data().src,
                   batch_size * sizeof(vid_t), cudaMemcpyDeviceToHost);

        cudaMemcpy(dst + peel_edges, hd_data().dst,
                   batch_size * sizeof(vid_t), cudaMemcpyDeviceToHost);

        #pragma omp parallel for
        for (uint32_t i = 0; i < batch_size; i++) {
            peel[peel_edges + i] = max_peel;
        }

        peel_edges += batch_size;
    }
    oper_bidirect_batch(hornet, hd_data().src, hd_data().dst, batch_size, DELETE);
}
```

Compared Against

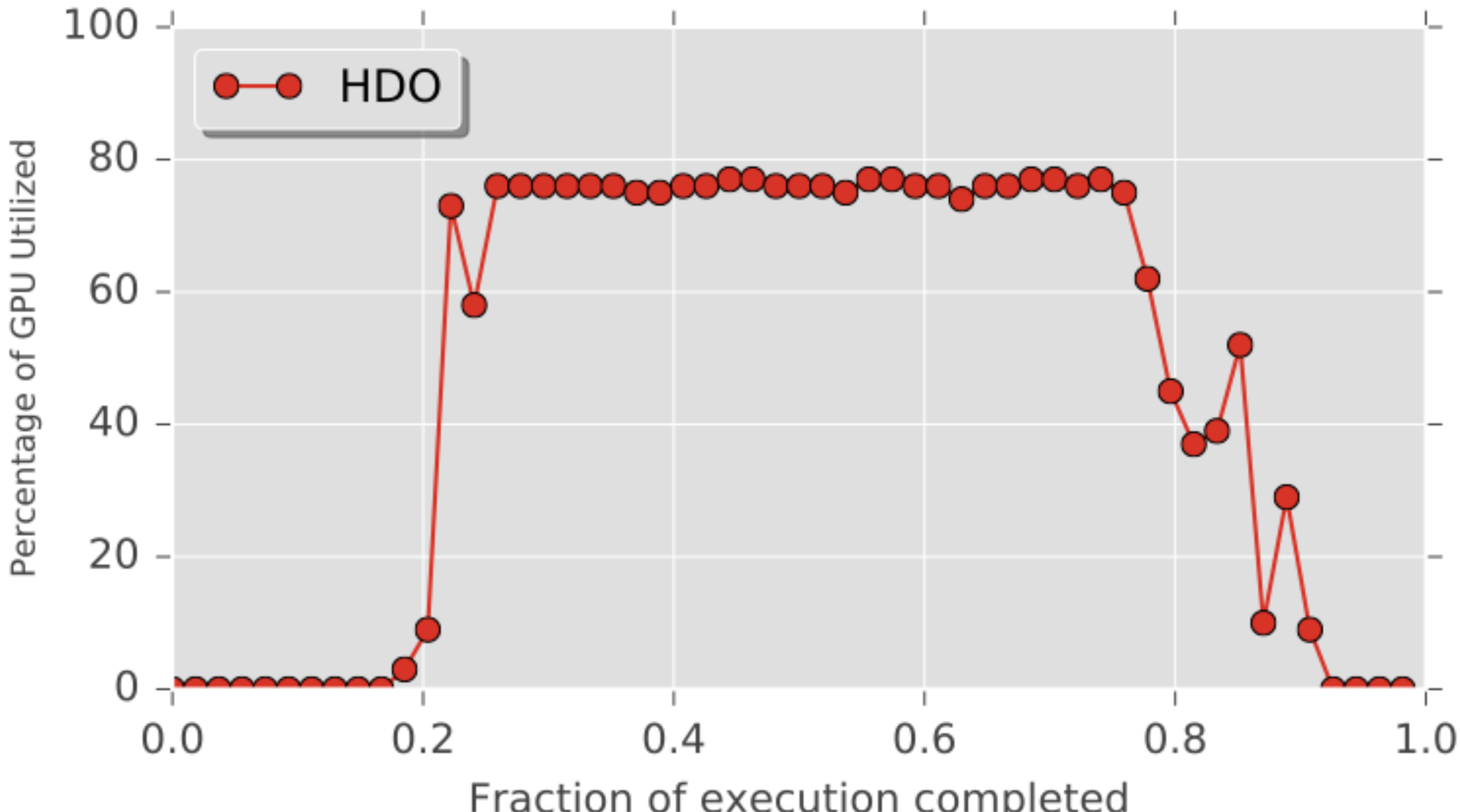
- ParK Extension
  - parallel k-core algorithm; IEEE BigData 2014
  - Some parallelism
  - No dynamic graph operations – vertex flagging
- igraph Extension
  - network analysis toolkit
  - Sequential
  - Uses edge deletions
- Both run on Intel Xeon E5-2695; 36 cores, 72 threads

k-core Decomp. Results

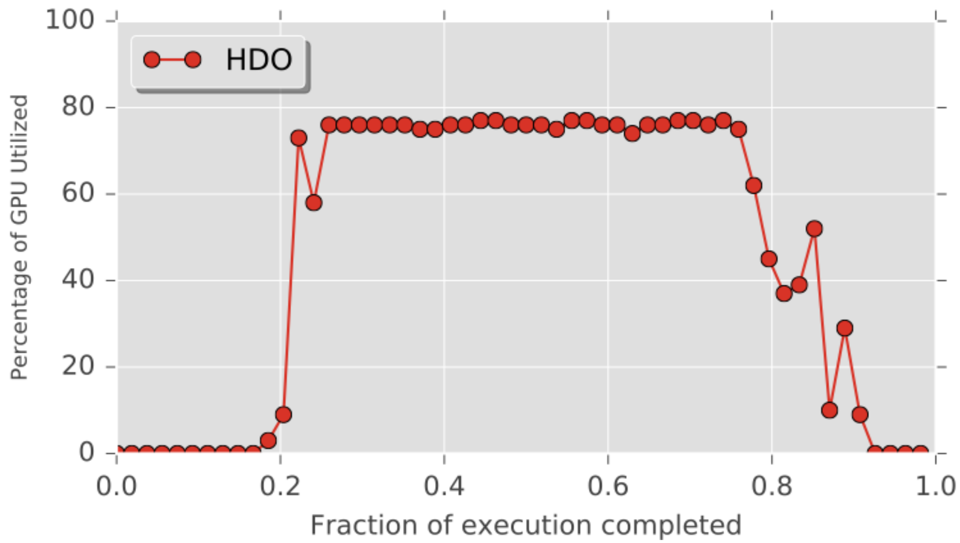
- Our algorithm always beats igraph
- Our algorithm always beats ParK ($1.2X - 7.8X$).
  - Usually $\sim 2X$ faster
- Our algorithm uses dynamic graph operations
  - And effectively uses the GPU

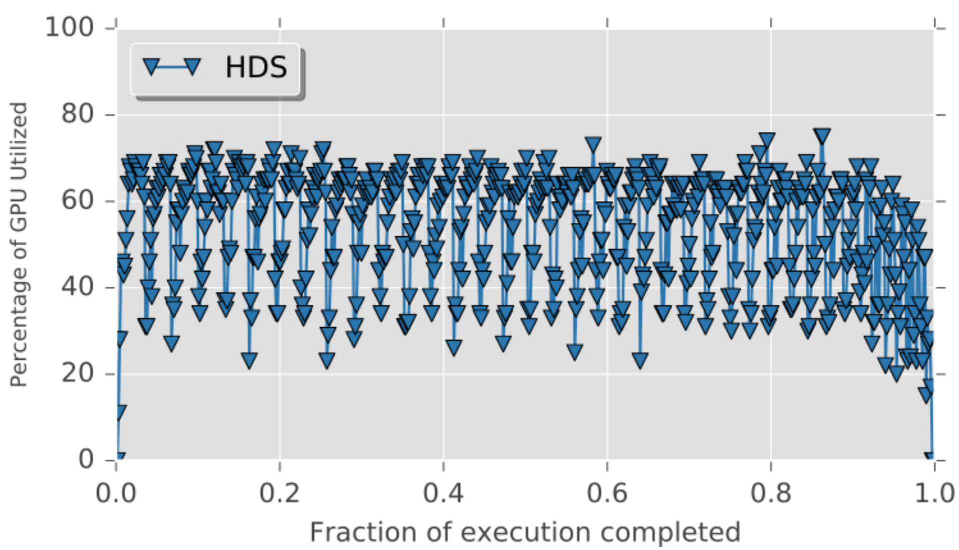
$Name$	$\|V\|$	$\|E\|$	$Our\ algorithm$	$ParK$	$igraph$
$dblp-author$	$5.5M$	$8.6M$	$129.2X$	$51.5X$	$1X$
$patentcite$	$3.8M$	$16.5M$	$63.8X$	$25X$	$1X$
$soc-LiveJournal1$	$4.8M$	$42.9M$	$25.9X$	$3.3X$	$1X$
$soc-pokec-relationships$	$1.6M$	$22.3M$	$85.9X$	$36.3X$	$1X$
$trackers$	$27.7M$	$140.6M$	$4.7X$	$4.1X$	$1X$

k-core Decomp. GPU Utilization



Decomp. vs. Slow Maximal k-core





Conclusion

- Dynamic graph operations can be computed on a GPU efficiently.
- Current idea:
  - Dynamic graph operations are only for dynamic graphs, not static graphs
- New idea: Static graph algorithms can benefit from dynamic graph operations
  - If we can efficiently utilize the system

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Takeaway

- Consider dynamic graph operations when you implement graph algorithms
  - Even if the graph doesn't change over time.

Thank you

Scalable K-Core Decomposition for Static Graphs Using a Dynamic Graph Data Structure




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- k-core Paper: Proceedings of IEEE BigData 2018
- k-truss, Hornet Paper: Proceedings of IEEE HPEC 2017/18
- Code: https://github.com/hornet-gt/hornet

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Backup slides

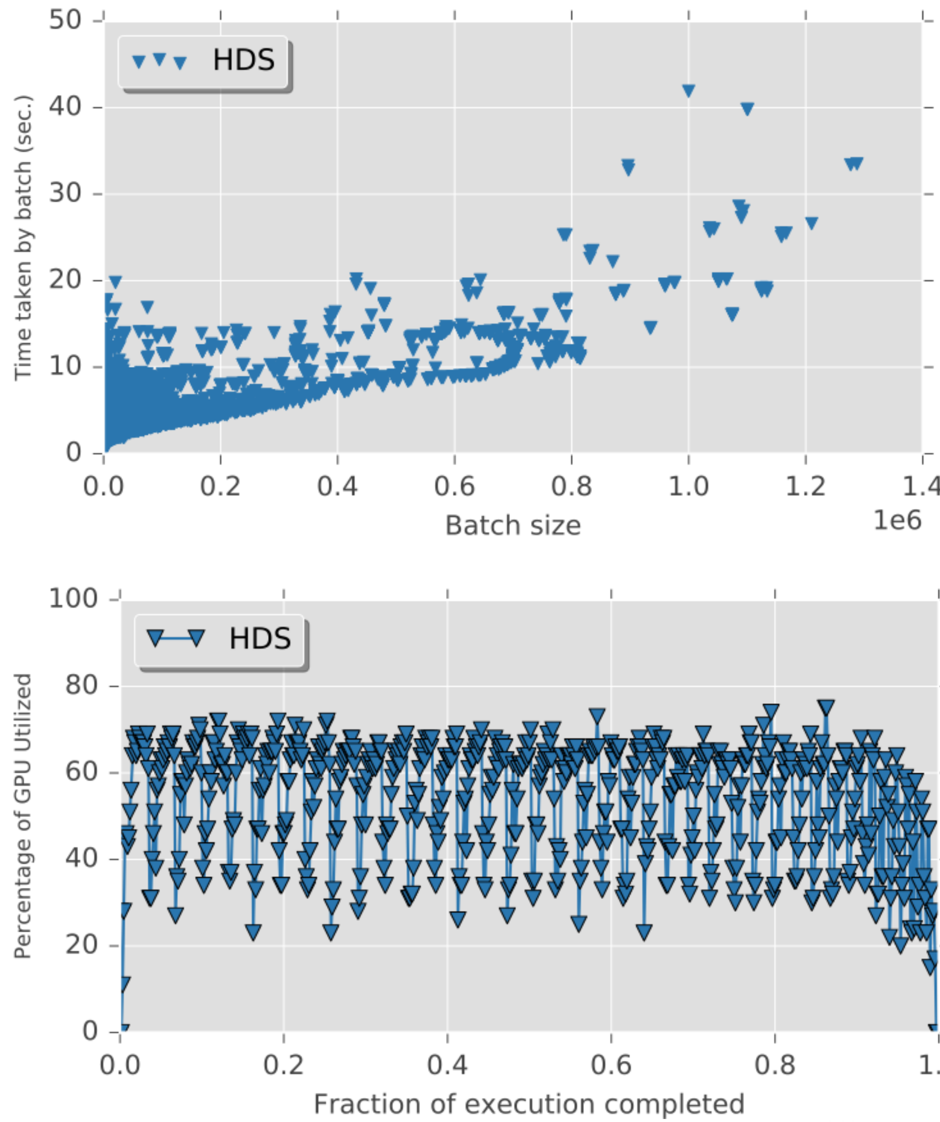
Performance

- Compared against
  - ParK: parallel k-core algorithm; BigData 2014
  - igraph: network analysis toolkit
- Dynamic graph data structure
  - Hornet, GPU-based
- Systems used
  - Our algorithms: NVIDIA P100
  - ParK, igraph: Intel Xeon E5-2695; 36 cores, 72 threads
    - igraph is sequential

Performance

- Compared against
  - Wang & Cheng: sequential algorithm for finding k-truss
  - Graphulo: parallel algorithm for finding k-tru
- Dynamic graph data structure
  - cuSTINGER-Delta, GPU-based
    - Evolved into Hornet
- Systems used
  - Our algorithm: NVIDIA P100
  - Wang & Cheng: Intel Core2 dual-core 2.80GHz CPU
  - Graphulo: 2 Intel i7dual-core

GPU Utilization / Batch Size



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HKS (maximal k-core) results

- ParK: k-core algorithm from IEEE Big Data 2014
- HKS run on NVIDIA P100 with Hornet data structure.

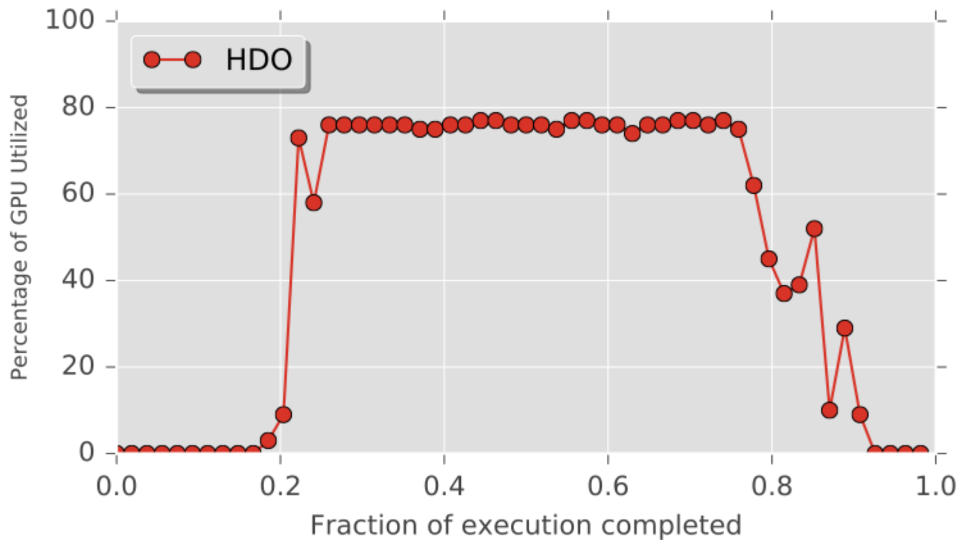
$Name$	$\|V\|$	$\|E\|$	$HKS\ (sec.)$	$ParK\ (sec.)$	$igraph\ (sec.)$
$dblp-author$	$5.5M$	$8.6M$	0.731 $2.2X$	0.105 $15X$	1.633 $1X$
$patentcite$	$3.8M$	$16.5M$	2.953 $1.3X$	0.253 $15X$	3.825 $1X$
$soc-LiveJournal1$	$4.8M$	$42.9M$	OOM OOM	0.549 $11.3X$	6.191 $1X$
$soc-pokec-relationships$	$1.6M$	$22.3M$	4.331 $0.6X$	0.155 $16.6X$	2.586 $1X$
$trackers$	$27.7M$	$140.6M$	OOM OOM	3.052 $6.8X$	20.693 $1X$
$wikipedia-link-de$	$3.2M$	$65.8M$	OOM OOM	0.764 $5.1X$	3.954 $1X$

HDS (k-core decomp) results

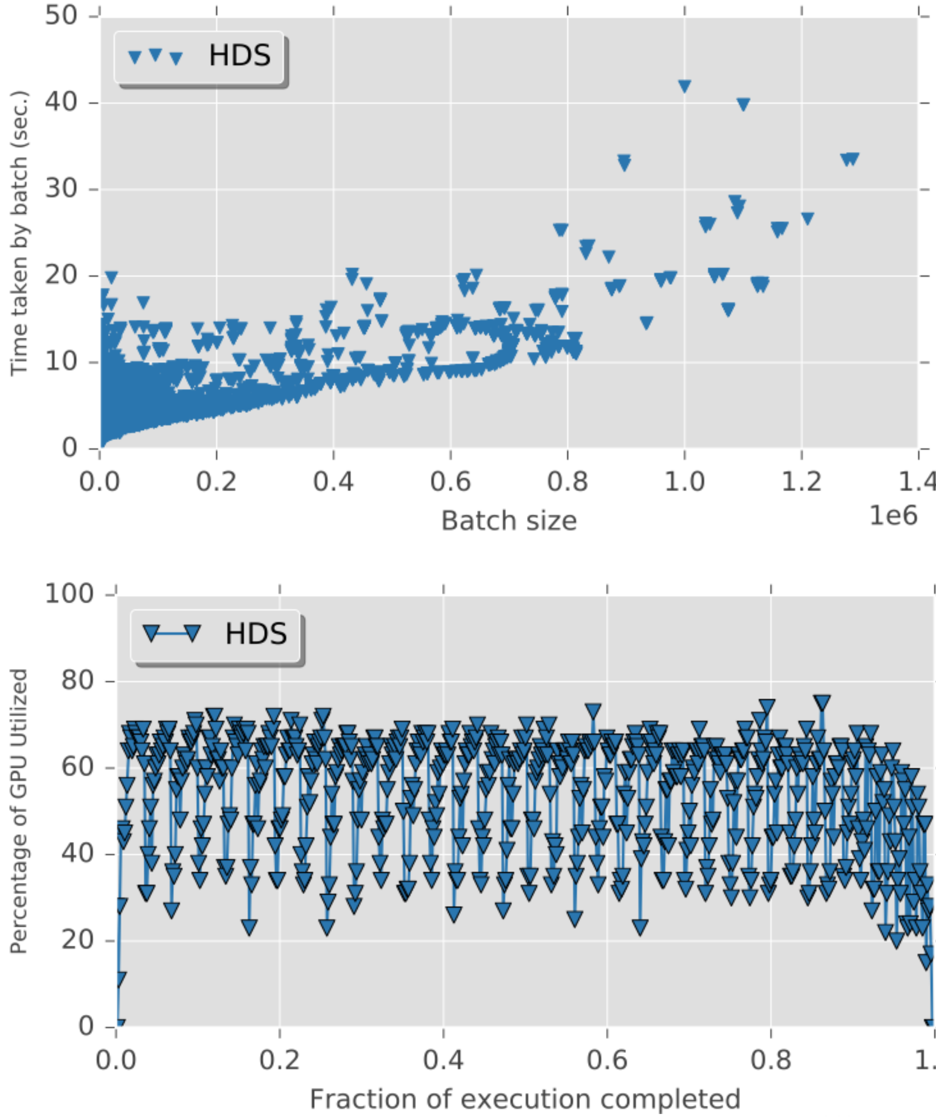
- ParK: k-core algorithm from IEEE Big Data 2014
- HDS run on NVIDIA P100 with Hornet data structure.

$Name$	$\|V\|$	$\|E\|$	$HDS\ (sec.)$	$ParK\ (sec.)$	$igraph\ (sec.)$
$dblp-author$	$5.5M$	$8.6M$	6.184 $13.3X$	1.595 $51.5X$	82.066 $1X$
$patentcite$	$3.8M$	$16.5M$	91.481 $3.6X$	13.294 $25X$	331.538 $1X$
$soc-LiveJournal1$	$4.8M$	$42.9M$	OOM OOM	487.112 $3.3X$	1572.985 $1X$
$soc-pokec-relationships$	$1.6M$	$22.3M$	50.049 $4.7X$	6.488 $36.3X$	235.790 $1X$
$trackers$	$27.7M$	$140.6M$	OOM OOM	1148.638 $4.1X$	4725.317 $1X$
$wikipedia-link-de$	$3.2M$	$65.8M$	OOM OOM	1397.323 $2.1X$	3003.166 $1X$

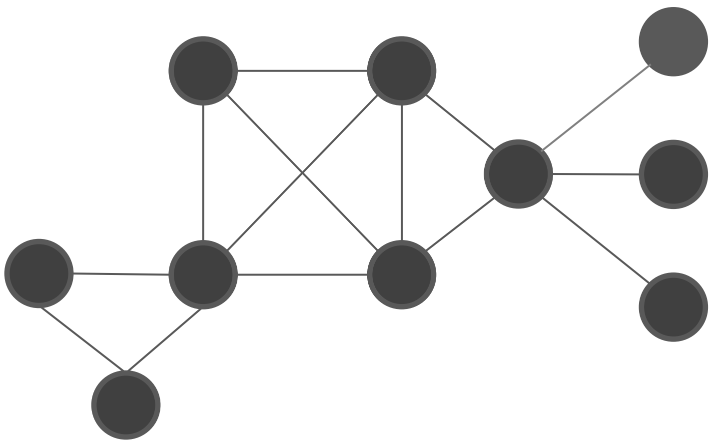
GPU Utilization



GPU Utilization / Batch Size



Maximal K-Core Algorithm (HKO)



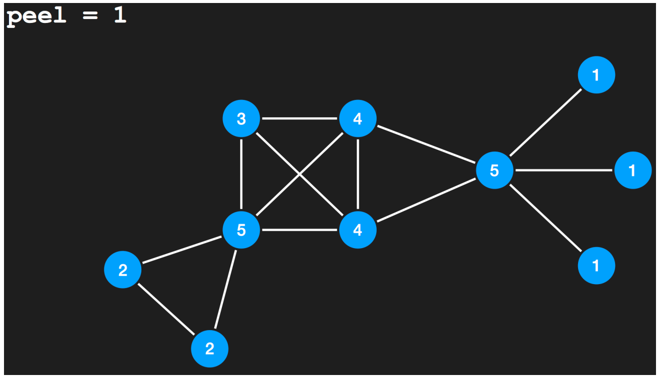
$peel = 3$

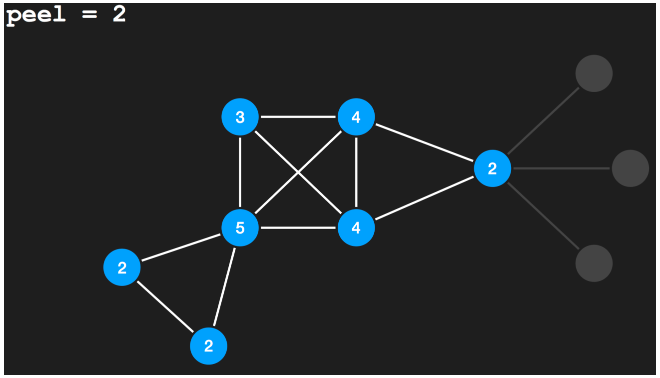
```
while there are non-flagged vertices

    flag all vertices with degree <= peel

    if there aren't any
        increment peel
    else
        for each flagged vertex v
            for each neighbor of v
                decrement neighbor's degree
```

Maximal K-Core Algorithm (HKO)

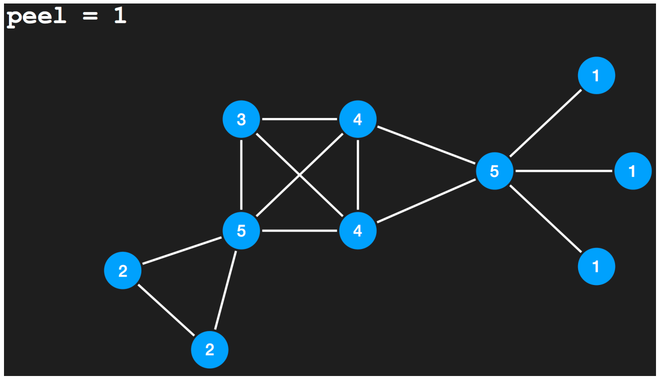


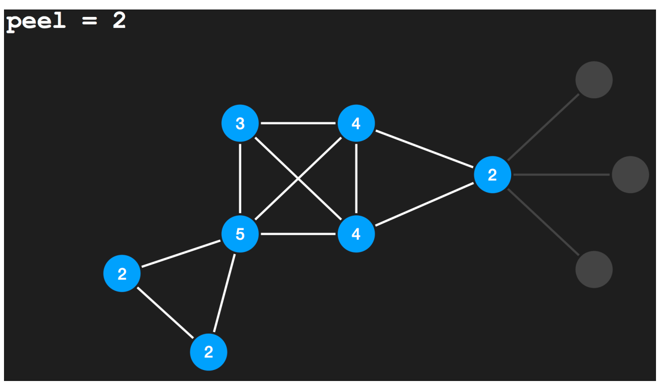


$peel \leftarrow 1$
$Q \leftarrow \{\}$
$num\_active = |V(G)|$
$color[v] \leftarrow 0 \forall v \in V(G)$
$deg[v] \leftarrow G.deg(v) \forall v \in V(G)$
while $num\_active > 0$ **do**
  $V_b \leftarrow \{\}$
  parallel for $v \in V(G) \wedge !flag[v]$ **do**
    if $deg[v] \leq peel$ **then**
      $flag[v] \leftarrow 1$
      $V_b.enqueue(v)$
  end parallel for
  $Q \leftarrow Q \cup V_b$
  $num\_active \leftarrow num\_active - |V_b|$

  if $|V_b| > 0$ **then**
    parallel for $(u, v) : u \in V_b, v \in adj(u)$ **do**
      $deg[u] \leftarrow deg[u] - 1$
      $deg[v] \leftarrow deg[v] - 1$
    end parallel for
  else
    $peel \leftarrow peel + 1$
    $Q \leftarrow \{\}$
return $(induced\_subgraph(G, Q), peel)$

Maximal K-Core Algorithm 1 (HKS)

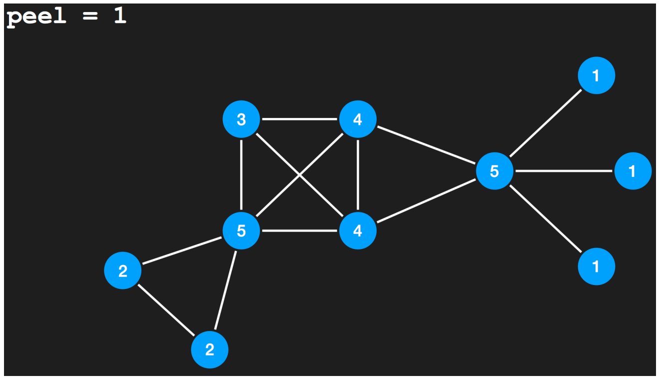


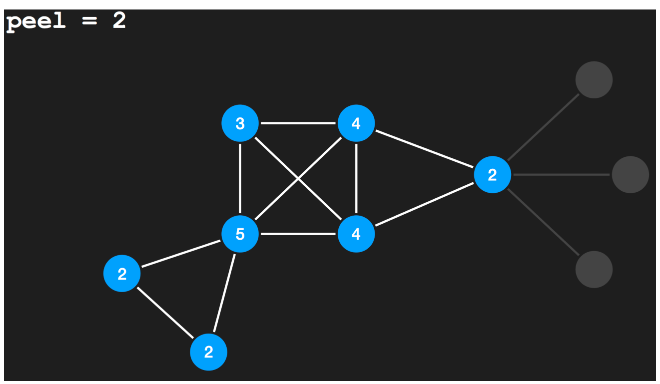



```
peel ← 1
Q ← {}
Ĝ ← ({}, {})
while |V(G)| > 0 do
    color[v] ← 0∀v ∈ V(G)
    V_b ← {}
    // Mark vertices with degree ≤ peel
    parallel for v ∈ V(G) do
        if deg[v] ≤ peel then
            color[v] ← 1
            V_b.enqueue(v)
    end parallel for

    if |V_b| > 0 then
        E_b ← {}
        // Mark edges with at least one marked vertex
        parallel for (u, v) : u ∈ V_b, v ∈ adj(u) do
            if color[u] or color[v] then
                E_b.enqueue((u, v))
        end parallel for
        // Delete these edges from G
        G.delete_edges(E_b)
        G.delete_vertices(V_b)
        // Insert these edges into Ĝ
        Ĝ.insert_vertices(V_b)
        Ĝ.insert_edges(E_b)
        Q ← Q ∪ V_b
    else
        peel ← peel + 1
        Q ← {}
return (induced_subgraph(Ĝ, Q), peel)
```

Maximal K-Core Algorithm 1 (HKS)





$peel \leftarrow 1$
$Q \leftarrow \{\}$
$\widehat{G} \leftarrow (\{\}, \{\})$
while $|V(G)| > 0$ **do**
  $color[v] \leftarrow 0 \forall v \in V(G)$
  $V_b \leftarrow \{\}$
  // Mark vertices with degree $\leq peel$
  parallel for $v \in V(G)$ **do**
    if $deg[v] \leq peel$ **then**
      $color[v] \leftarrow 1$
      $V_b.enqueue(v)$
  end parallel for

  if $|V_b| > 0$ **then**
    $E_b \leftarrow \{\}$
    // Mark edges with at least one marked vertex
    parallel for $(u, v) : u \in V_b, v \in adj(u)$ **do**
      if $color[u]$ or $color[v]$ **then**
        $E_b.enqueue((u, v))$
    end parallel for
    // Delete these edges from G
    $G.delete\_edges(E_b)$
    $G.delete\_vertices(V_b)$
    // Insert these edges into $\widehat{G}$
    $\widehat{G}.insert\_vertices(V_b)$
    $\widehat{G}.insert\_edges(E_b)$
    $Q \leftarrow Q \cup V_b$
  else
    $peel \leftarrow peel + 1$
    $Q \leftarrow \{\}$
return $(induced\_subgraph(\widehat{G}, Q), peel)$

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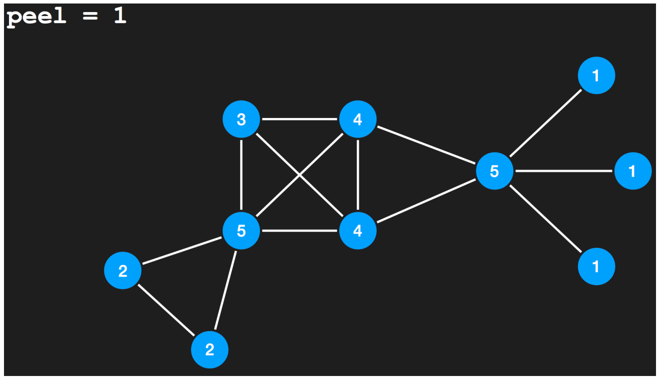
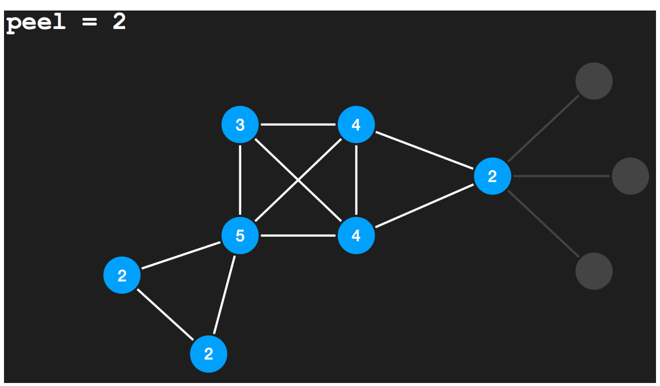
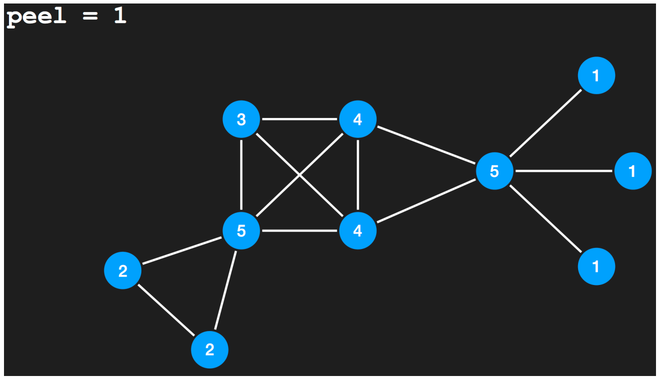
Maximal K-Core Algorithm 1 (HKS)

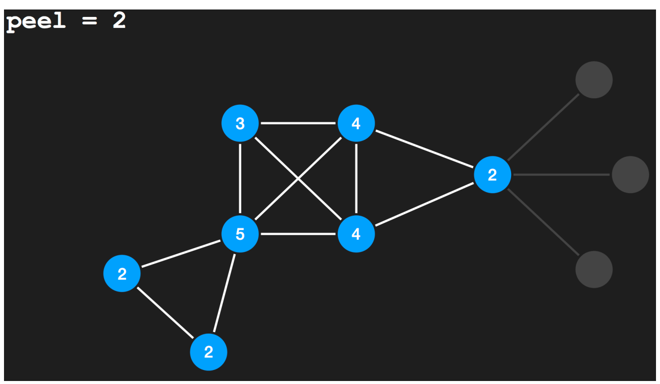





```
peel ← 1
Q ← {}
Ĝ ← ({}, {})
while |V(G)| > 0 do
    color[v] ← 0 ∀v ∈ V(G)
    V_b ← {}
    // Mark vertices with degree ≤ peel
    parallel for v ∈ V(G) do
        if deg[v] ≤ peel then
            color[v] ← 1
            V_b.enqueue(v)
    end parallel for
    if |V_b| > 0 then
        E_b ← {}
        // Mark edges with at least one marked vertex
        parallel for (u, v) : u ∈ V_b, v ∈ adj(u) do
            if color[u] or color[v] then
                E_b.enqueue((u, v))
        end parallel for
        // Delete these edges from G
        G.delete_edges(E_b)
        G.delete_vertices(V_b)
        // Insert these edges into Ĝ
        Ĝ.insert_vertices(V_b)
        Ĝ.insert_edges(E_b)
        Q ← Q ∪ V_b
    else
        peel ← peel + 1
        Q ← {}
return (induced_subgraph(Ĝ, Q), peel)
```

Maximal K-Core Algorithm 1 (HKS)

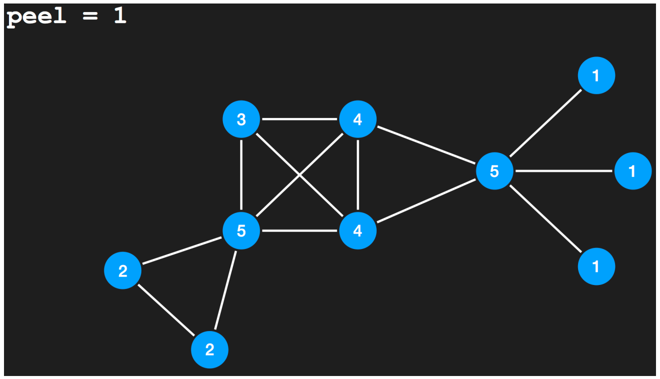


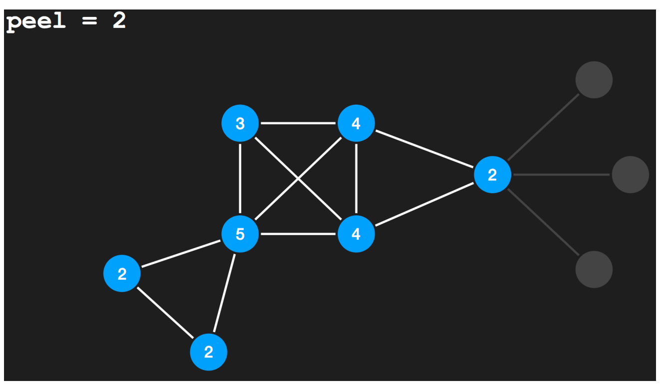


$peel \leftarrow 1$
$Q \leftarrow \{\}$
$\widehat{G} \leftarrow (\{\}, \{\})$
while $|V(G)| > 0$ **do**
  $color[v] \leftarrow 0 \forall v \in V(G)$
  $V_b \leftarrow \{\}$
  // Mark vertices with degree $\leq peel$
  parallel for $v \in V(G)$ **do**
    if $deg[v] \leq peel$ **then**
      $color[v] \leftarrow 1$
      $V_b.enqueue(v)$
  end parallel for

  if $|V_b| > 0$ **then**
    $E_b \leftarrow \{\}$
    // Mark edges with at least one marked vertex
    parallel for $(u, v) : u \in V_b, v \in adj(u)$ **do**
      if $color[u]$ or $color[v]$ **then**
        $E_b.enqueue((u, v))$
    end parallel for
    // Delete these edges from G
    $G.delete\_edges(E_b)$
    $G.delete\_vertices(V_b)$
    // Insert these edges into $\widehat{G}$
    $\widehat{G}.insert\_vertices(V_b)$
    $\widehat{G}.insert\_edges(E_b)$
    $Q \leftarrow Q \cup V_b$
  else
    $peel \leftarrow peel + 1$
    $Q \leftarrow \{\}$
return $(induced\_subgraph(\widehat{G}, Q), peel)$

Maximal K-Core Algorithm 1 (HKS)





$peel \leftarrow 1$
$Q \leftarrow \{\}$
$\widehat{G} \leftarrow (\{\}, \{\})$
while $|V(G)| > 0$ **do**
  $color[v] \leftarrow 0 \forall v \in V(G)$
  $V_b \leftarrow \{\}$
  // Mark vertices with degree $\leq peel$
  parallel for $v \in V(G)$ **do**
    if $deg[v] \leq peel$ **then**
      $color[v] \leftarrow 1$
      $V_b.enqueue(v)$
  end parallel for

  if $|V_b| > 0$ **then**
    $E_b \leftarrow \{\}$
    // Mark edges with at least one marked vertex
    parallel for $(u, v) : u \in V_b, v \in adj(u)$ **do**
      if $color[u]$ or $color[v]$ **then**
        $E_b.enqueue((u, v))$
    end parallel for
    // Delete these edges from G
    $G.delete\_edges(E_b)$
    $G.delete\_vertices(V_b)$
    // Insert these edges into $\widehat{G}$
    $\widehat{G}.insert\_vertices(V_b)$
    $\widehat{G}.insert\_edges(E_b)$
    $Q \leftarrow Q \cup V_b$
  else
    $peel \leftarrow peel + 1$
    $Q \leftarrow \{\}$
return $(induced\_subgraph(\widehat{G}, Q), peel)$

K-Core Decomp. Algorithm 1 (HDS)

$\widehat{G} \leftarrow (\{\}, \{\})$
while $|V(G)| > 0$ **do**
    // Find maximal k-core of G
    $K, k\_num \leftarrow KcoreNum1(G, \widehat{G})$
    // Mark edges in the maximal k-core with the *peel* number
    parallel for $e \in E(K)$ **do**
        $peels[e] \leftarrow k\_num$
    end parallel for
    // Delete the k-core edges and vertices
    $\widehat{G}.delete\_edges(E(K))$
    $\widehat{G}.delete\_vertices(V(K))$
    $swap(G, \widehat{G})$
return $peels[]$

HKO (maximal k-core) results

- ParK: k-core algorithm from IEEE Big Data 2014
- HKO run on NVIDIA P100 with Hornet data structure.

$Name$	$\|V\|$	$\|E\|$	$HKO\ (sec.)$	$ParK\ (sec.)$	$igraph\ (sec.)$
$dblp - author$	$5.5M$	$8.6M$	0.028 $15X$	0.105 $15X$	1.633 $1X$
$patentcite$	$3.8M$	$16.5M$	0.147 $26X$	0.253 $15X$	3.825 $1X$
$soc - LiveJournal1$	$4.8M$	$42.9M$	0.838 $7.4X$	0.549 $11.3X$	6.191 $1X$
$soc - pokec - relationships$	$1.6M$	$22.3M$	0.174 $15X$	0.155 $16.6X$	2.586 $1X$
$trackers$	$27.7M$	$140.6M$	13.160 $1.6X$	3.052 $6.8X$	20.693 $1X$
$wikipedia - link - de$	$3.2M$	$65.8M$	1.987 $2X$	0.764 $5.1X$	3.954 $1X$

HDO (k-core decomp) results

- ParK: k-core algorithm from IEEE Big Data 2014
- HDO run on NVIDIA P100 with Hornet data structure.

$Name$	$\|V\|$	$\|E\|$	$HDO\ (sec.)$	$ParK\ (sec.)$	$igraph\ (sec.)$
$dblp - author$	$5.5M$	$8.6M$	0.635 $129.2X$	1.595 $51.5X$	82.066 $1X$
$patentcite$	$3.8M$	$16.5M$	5.200 $63.8X$	13.294 $25X$	331.538 $1X$
$soc - LiveJournal1$	$4.8M$	$42.9M$	60.755 $25.9X$	487.112 $3.3X$	1572.985 $1X$
$soc - pokec - relationships$	$1.6M$	$22.3M$	2.756 $85.9X$	6.488 $36.3X$	235.790 $1X$
$trackers$	$27.7M$	$140.6M$	1006.954 $4.7X$	1148.638 $4.1X$	4725.317 $1X$
$wikipedia - link - de$	$3.2M$	$65.8M$	266.923 $11.3X$	1397.323 $2.1X$	3003.166 $1X$